Trinity College

Semester One Examination, 2016

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	100	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

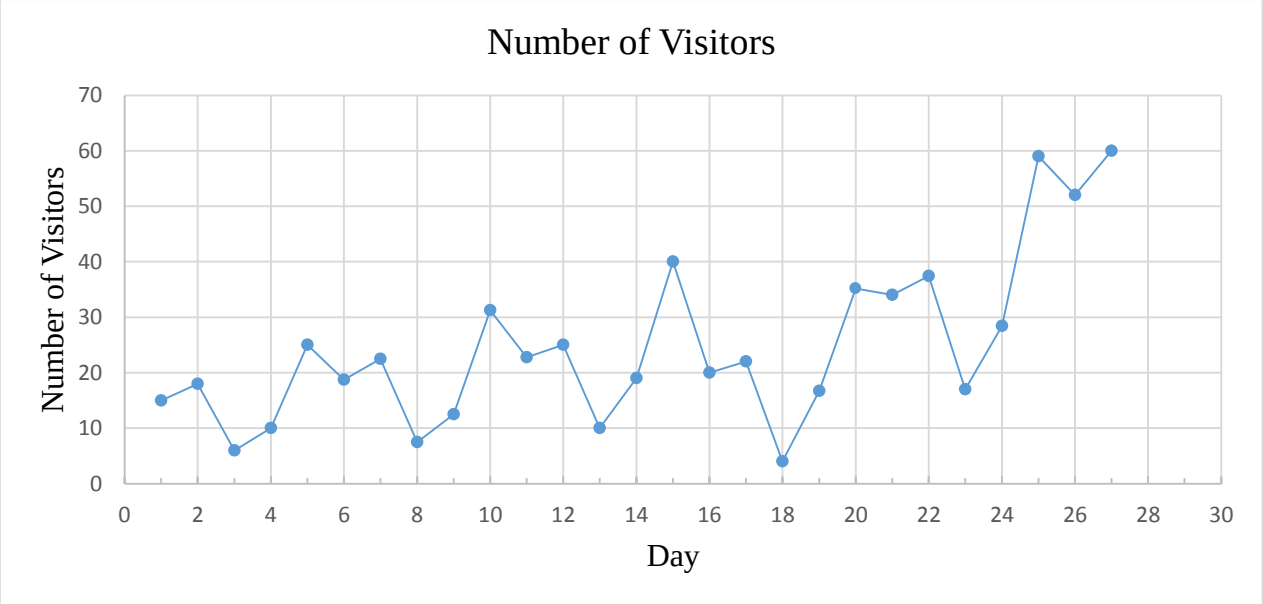
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **65% (100 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(4 marks)**



(a) Suggest, with reasons, an appropriate moving average to smooth the given set of data. (2 marks)

Solution
A 5point moving average because there is a peak every 5 seasons.
Specific behaviours
✓ correct Moving average ✓ correct reasoning

(b) Describe long term trend in this time series. (1 mark)

Solution
The long term trend is increasing
Specific behaviours
✓ states increasing

(c) Describe any unusual fluctuations in this time series. (1 mark)

Solution
Day 18 has an unusually low value.
Specific behaviours
✓ states day 18 and that it is low

Question 9 **(8 marks)**

(a) Describe a suitable method to organise and display data when investigating the existence of an association between two categorical variables. (2 marks)

Solution
A two-way frequency table, with either row or column percentages. Might also choose to construct a stacked bar graph.
Specific behaviours
✓ states frequency table ✓ indicates need to calculate row or column percentages

(b) A class was set a task to investigate whether an association exists between the distance a student lived from school and the number of times they were late in a term.

(i) What **type** of graph would be appropriate to display data collected? (1 mark)

Solution
A scatterplot
Specific behaviours
✓ states graph type

(ii) What statistical measure would be useful to calculate in order to determine whether an association existed? (1 mark)

Solution
Correlation coefficient
Specific behaviours
✓ states measure

(iii) One student designed the questionnaire shown below. Comment on the appropriateness of their design for this investigation. (2 marks)

Name:

Tick one box	Distance less than 2 km	Distance more than 2 km
Late less than 3 times		
Late more than 3 times		

Solution
Not very appropriate or useful - Better to record exact distances and number of lates for each person - Late group boundary doesn't allow for 3 lates, etc
Specific behaviours
✓ comments that form not good ✓ supplies reasons

(iv) A student carried out the investigation, found that a moderate negative association existed, and concluded that frequent lateness was caused by living close to the school. Comment on their conclusion. (2 marks)

Solution
Conclusion implies that one causes the other, which may not be true. All that can be concluded is that an association exists between the variables.
Specific behaviours
✓ disagrees with conclusion ✓ notes causation implied

Question 10 **(7 marks)**

The number of votes still to count at the end of an election decreased by 72 every minute after 6 pm. At 6 pm, 2955 votes still needed counting.

(a) Show that by 6:02 pm, 2811 votes still needed counting. (1 mark)

Solution
$2955 - 2 \times 72 = 2811$ votes
Specific behaviours
✓ shows calculation

(b) Deduce a non-recursive rule for T_n, the number of votes still needing counting n minutes after 6 pm. (2 marks)

Solution
$T_n = 2955 - 72n$
Specific behaviours
✓ uses initial number and difference ✓ states correct rule

(c) Determine how many votes still needed counting at 6:30 pm. (1 mark)

Solution
$2955 - 30 \times 72 = 795$ votes
Specific behaviours
✓ determines number of votes

(d) At 6:30 pm, counting slowed so that only 36 votes were processed every minute. Determine the time, to the nearest minute, that counting finished. (3 marks)

Solution
$T_n = 795 - 36n$ $795 - 36n = 0 \Rightarrow n = 22.08$ $30 + 22 = 52 \Rightarrow$ Time will be 6:52 pm
Specific behaviours
✓ states new rule ✓ solves for n ✓ determines correct time of day

Question 11 **(17 marks)**

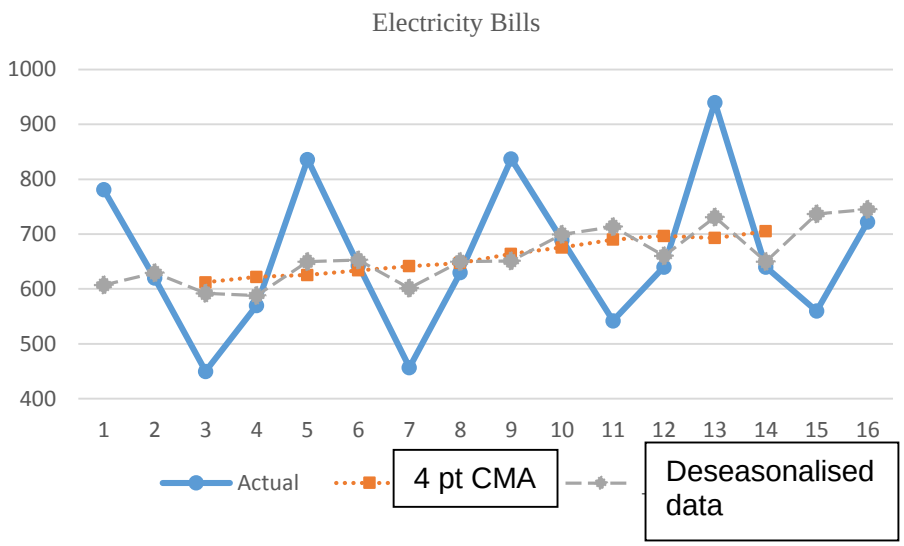
The table below shows the quarterly electricity bill for a 5 person household over a four year time period.

Time (t)	Year	Quarter	Bill ($)	4 pt. CMA	Bill as a % of Quarter
1	2012	Q1	781		129.04%
2		Q2	620		102.44%
3		Q3	450	612.13	74.35%
4		Q4	570	621.88	94.18%
5	2013	Q1	836	625.63	130.32%
6		Q2	643	634.00	a
7		Q3	457	b	71.24%
8		Q4	630	647.50	98.21%
9	2014	Q1	837	663.88	123.63%
10		Q2	689	675.75	101.77%
11		Q3	c	689.88	80.06%
12		Q4	640	696.63	94.53%
13	2015	Q1	940	692.75	131.38%
14		Q2	640	705.25	89.45%
15		Q3	560		78.27%
16		Q4	722		100.91%

(a) Determine the values of a, b and c in the table. (3 marks)

Solution		
a = 100.23%	b = 641.63	c = 540
Specific behaviours		
✓ correctly identifies values		

(b) Fill in the two missing labels on the graph below. (2 marks)



(c) With reference to the graph, explain whether the 4 point centred moving average or deseasonalised data does a better job of smoothing the time series. (2 marks)

Solution
The 4 point centred moving average is better because there is less variation. (Closer to linear)
Specific behaviours
✓ correctly identifies 4 PT CMA ✓ correctly states why

(d) Determine the seasonal quarterly index for Q1. (2 marks)

Solution
$\frac{129.04 + 130.32 + 123.63 + 131.38}{4} = 128.59\%$ $= 1.2859$
Specific behaviours
✓ correctly calculates values ✓ states answer as decimal number

(e) Comment on the cost of the Q1 bills in relation the average quarterly bill. (2 marks)

Solution
Q1 is generally 29% above the average bill.
Specific behaviours
✓ states above average ✓ uses percentage

(f) Describe any long term fluctuations (cycles) in this time series. (1 marks)

Solution
The bills are increasing over time
Specific behaviours
✓ states increasing

(g) Determine the equation of the regression line $M = A + Bt$. (2 marks)

Solution
$\hat{M} = 582.9 + 8.9419t$
Specific behaviours
✓ correct A value ✓ correct B value

(h) Use your regression line from (g) to predict the value of the Q1 bill in 2017. (3 marks)

Solution
$t = 21$ $\hat{M} = [5.829 + 8.9419(21)] \times 1.2859$ $= 991.017$ $\approx \$991$
Specific behaviours
✓ $t = 21$ ✓ multiplies by seasonal index ✓ correctly states answer

Question 12 **(8 marks)**

A media company sought responses from the general public to the question "*How much trust do you have in the following for information about asylum seekers?".* The company were investigating whether the source of information was associated with the degree of trust the general public placed in the information about asylum seekers.

The responses are summarised in the table below.

	Degree of trust in asylum seeker information		
Information source	Some trust	Little trust	Not sure
Politicians	27	117	16
The media	25	84	11
Doctors	99	61	20
Churches	54	80	16

(a) Name the explanatory and response variables for this investigation. (2 marks)

Solution
Explanatory is SOURCE and response is DEGREE OF TRUST
Specific behaviours
✓ states correct explanatory variable ✓ states correct response variable

(b) Complete the table of percentages below, rounding to the nearest whole number, so that it can be used to identify whether the source of information is associated with the degree of trust the general public place in the information. (4 marks)

	Degree of trust in information		
Information source	Some trust	Little trust	Not sure
Politicians	**17**	**73**	**10**
The media	**21**	**70**	**9**
Doctors	**55**	**34**	**11**
Churches	**36**	**53**	**11**

Solution
See table - using row totals of 160, 120, 180 and 150.
Specific behaviours
✓ calculates row totals ✓ calculates one row of percentages ✓ rounds to whole numbers ✓ completes all row percentages

(c) Comment on whether this data provides any evidence that the source of information is associated with the degree of trust placed in the information about asylum seekers. (2 marks)

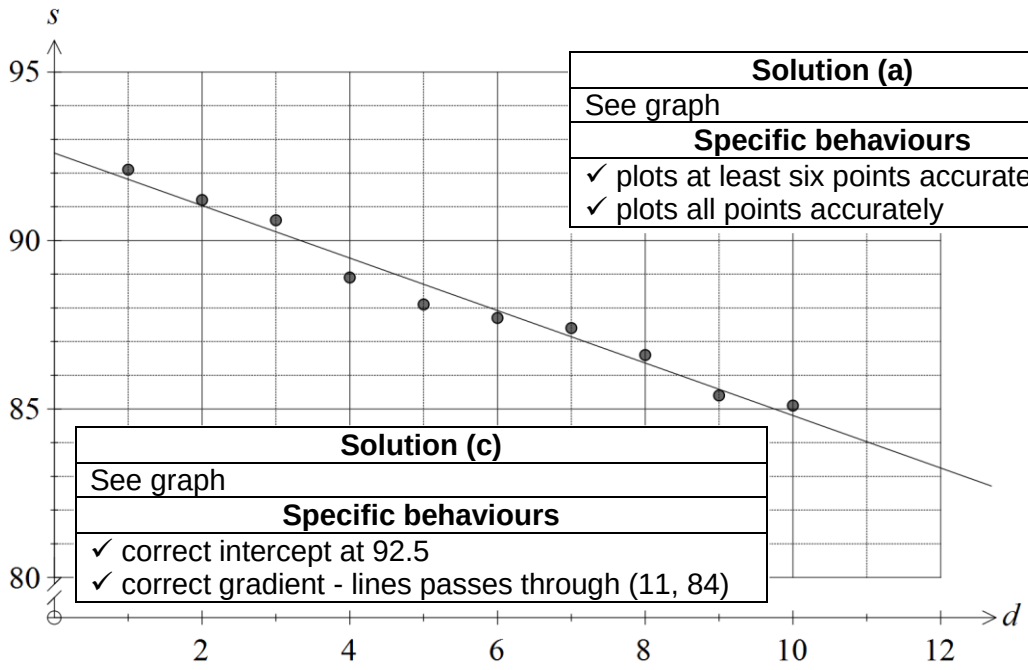
Solution
Yes - an association exists. Only 17% have some trust when the information source is a politician compared to 55% when the source is a doctor.
Specific behaviours
✓ States that evidence exists ✓ uses an example to support claim

Question 13 **(8 marks)**

The daily customer satisfaction index was measured by an online business over a period of ten consecutive days and the data collected is shown in the table below.

Day (d)	1	2	3	4	5	6	7	8	9	10
CS Index (s)	92.1	91.2	90.6	88.9	88.1	87.7	87.4	86.6	85.4	85.1

(a) Plot the above data on the axes below. (2 marks)



Solution (a)
See graph
Specific behaviours
✓ plots at least six points accurately ✓ plots all points accurately

Solution (c)
See graph
Specific behaviours
✓ correct intercept at 92.5 ✓ correct gradient - lines passes through (11, 84)

(b) Determine the equation of the least-squares line that models the linear relationship between the day number and the customer satisfaction index. (2 marks)

Solution
$s = -0.779d + 92.59$
Specific behaviours
✓ determines gradient and intercept ✓ uses correct variables

(c) Draw the least-squares line on the axes above. (2 marks)

(d) Predict the customer satisfaction index for day 11. (1 mark)

Solution
$s = 84.0$
Specific behaviours
✓ calculates value

(e) Explain why a prediction for the customer satisfaction index for day 15 should be treated with caution. (1 mark)

Solution
Prediction involves considerable extrapolation.
Specific behaviours
✓ comments on extrapolation

Question 14 **(9 marks)**

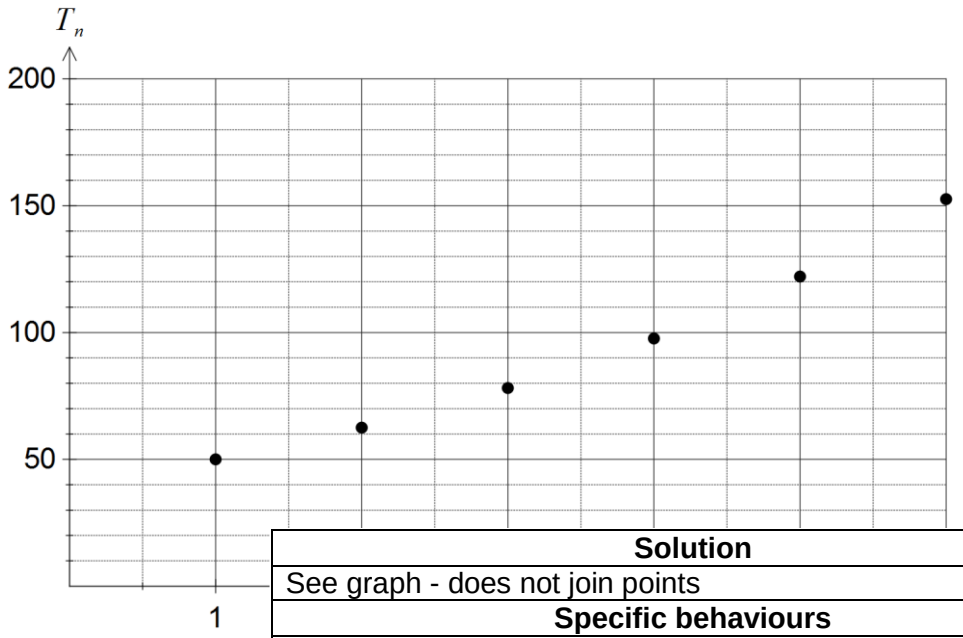
Sequence T is defined given by $T_{n+1} = 1.25T_n,\ T_1 = 50$.

(a) Use the recursive rule to complete the table below, rounding values to one decimal place. (2 marks)

n	1	2	3	4	5	6
T_n	50	**62.5**	**78.1**	**97.7**	**122.1**	**152.6**

Solution
See table
Specific behaviours
✓ calculates at least three correct values ✓ calculates all correct values

(b) Graph the first six terms of sequence T on the axes below. (2 marks)



Solution
See graph - does not join points
Specific behaviours
✓ plots three points accurately ✓ plots all points accurately and does not attempt to join them

The first three terms of the geometric sequence U are 200, 160 and 128.

(c) Deduce a rule for the n^{th} term of sequence U. (2 marks)

Solution
$r = \frac{160}{200} = 0.8$ $U_n = 200(0.8)^{n-1}$
Specific behaviours
✓ calculates ratio of terms ✓ writes general rule for n^{th} term

(d) Determine U_{10}. (1 mark)

Solution
$U_{10} = 200 \times 0.8^9 \approx 26.8$ (1dp)
Specific behaviours
✓ calculates term

(e) Determine the largest value of *n* so that $U_n > T_n$, justifying your answer. (2 marks)

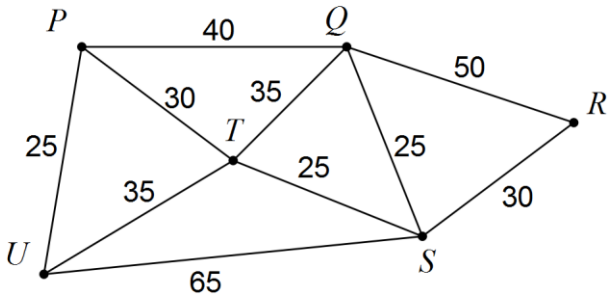
Solution
$n = 4$ $T_4 = 97.6,\ U_4 = 102.4 \Rightarrow U_n > T_n$ $T_5 = 122.1,\ U_5 = 81.92 \Rightarrow U_n < T_n$
Specific behaviours
✓ states correct value of *n* ✓ justifies answer

Question 15 **(9 marks)**

A business has branches in six cities. The table below shows the time, in minutes, it takes for a package received at one branch to be transported to a branch in another city, where a direct route exists.

Q	40				
R	-	50			
S	-	25	30		
T	30	35	-	25	
U	25	-	-	65	35
	P	Q	R	S	T

(a) Construct a weighted graph to show this information, using the cities placed below. (3 marks)



Solution
See diagram
Specific behaviours
✓ correctly adds at least 7 edges ✓ adds all edges correctly ✓ labels all edges correctly

(b) Determine the shortest transport time for a package to travel from

(i) P to S. (1 mark)

Solution
55 minutes
Specific behaviours
✓ states correct time

(ii) Q to U. (1 mark)

Solution
65 minutes
Specific behaviours
✓ states correct time

(c) A document needs to be sent from branch U via branch R, where a customer will sign the document, to branch P. Determine the minimum transport time for the document to make this journey, listing all branches on the way. (2 marks)

Solution
U - T - S - R - S - T - P = 175 minutes
Specific behaviours
✓ states minimum time ✓ lists branches

(d) Another business document requires signing by the manager of each branch. In planning a route for this document, would finding a Eulerian trail be more appropriate than finding a Hamiltonian trail? Explain your answer. (2 marks)

Solution
No. Hamiltonian is needed, as every vertex must be visited just once.
Specific behaviours
✓ Answers no ✓ Explains Hamiltonian trail

Question 16 **(7 marks)**

A fish farmer initially stocked a tank with 50 small fish. At the end of each month, the farmer caught some of the largest fish and sold them before adding more, smaller fish to the tank.

The number of fish in the tank at the start of the n^{th} month is given by F_n, where $F_{n+1} = 0.7F_n + 120, \quad F_1 = 50$.

(a) Use the recurrence relation to state

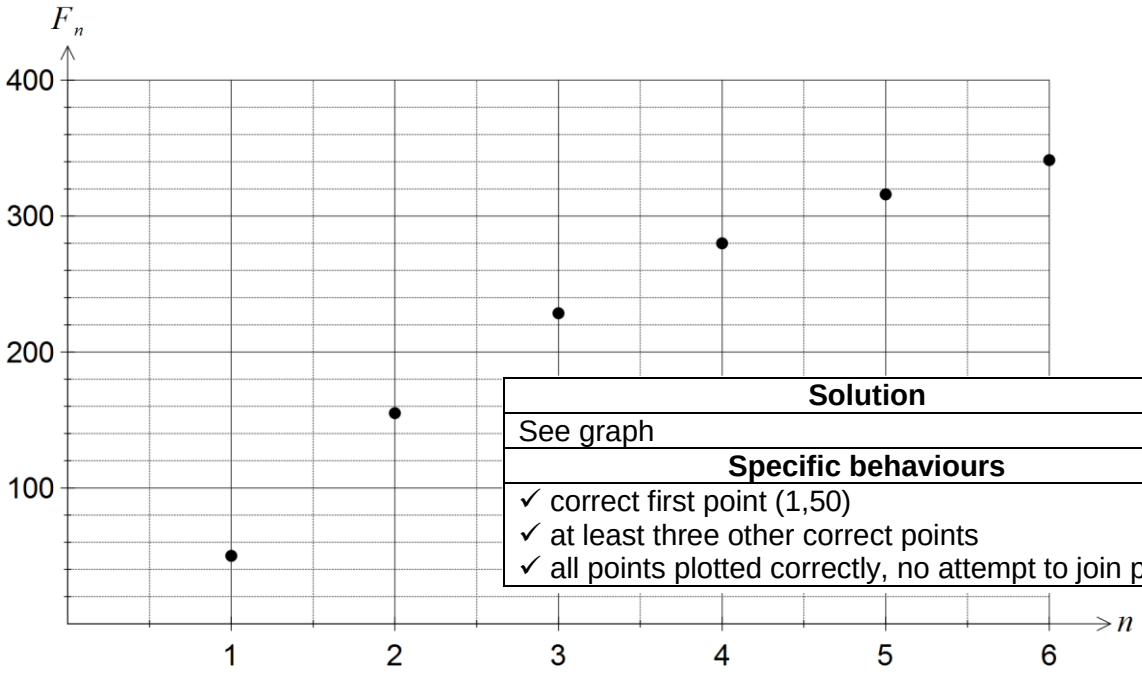
(i) the number of smaller fish added to the tank each month. (1 mark)

Solution
120
Specific behaviours
✓ states value

(ii) the percentage of the fish caught and sold each month. (1 mark)

Solution
30% *(NB 100%-70% caught each month)*
Specific behaviours
✓ states value

(b) Graph F_n on the axes below for $1 \le n \le 6$. (3 marks)



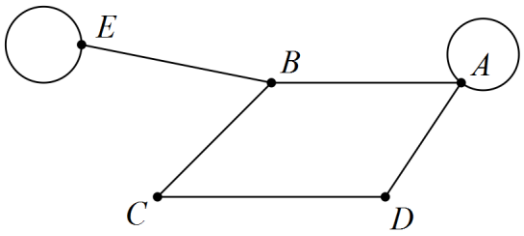
Solution
See graph
Specific behaviours
✓ correct first point (1,50) ✓ at least three other correct points ✓ all points plotted correctly, no attempt to join points

(c) Assuming this model continues, comment on how the number of fish in the tank changes over the next few years. (2 marks)

Solution
Each month, number of fish increase but at a slower rate, until eventually reach a steady state of 400 fish in tank at start of each month.
Specific behaviours
✓ comments on increasing at slower rate ✓ determines long term steady state of 400 fish

Question 17 **(9 marks)**

An airline has flights between six cities as shown in the graph below. Two of the flights are sightseeing flights that return to the city from which they departed.



(a) Determine M, the adjacency matrix for this graph. (3 marks)

Solution
$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$
Specific behaviours
✓ draws a 5x5 matrix ✓ completes at least three correct rows ✓ completes all rows correctly

(b) Calculate M^2 and explain the significance of the elements in this matrix that are zero. (3 marks)

Solution
$M^2 = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 \\ 1 & 3 & 0 & 2 & 1 \\ 2 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 0 & 2 \end{bmatrix}$ A zero indicates that not possible to travel between these cities by taking exactly two flights.
Specific behaviours
✓ shows M^2 is a 5x5 matrix ✓ calculates matrix correctly ✓ explains zeros

(c) Determine the number of zero elements in the matrix $M + M^2$ and explain their significance in terms of specific flight(s). (3 marks)

Solution
There are two zeros, $M_{4,5}$ and $M_{5,4}$. There is no way to travel between D and E taking either one or two flights.
Specific behaviours
✓ states number of zeros ✓ states cities involved ✓ states significance

Question 18 **(6 marks)**

From observations of a random sample of 236 blackbirds, the equation of the least-squares line that models the relationship between the wing span (s, measured in centimetres) and the mass (m, measured in grams) of blackbirds was found to be $s = 0.085m + 28.4$. The coefficient of determination between the variables was 0.79.

(a) State the percentage of the variation in wing span of blackbirds that can be explained by the variation in their mass. (1 marks)

Solution
79%
Specific behaviours
✓ states percentage

(b) Calculate the correlation coefficient between s and m, using the fact that the direction of the association is positive. (1 marks)

Solution
$r^2 = 0.79$ $r = \pm\sqrt{0.79} \Rightarrow r \approx 0.89, -0.89$ (2dp) +ve assn $\Rightarrow r \approx 0.89$
Specific behaviours
✓ calculates two possible values ✓ eliminates negative value

(c) Predict the wing span of a blackbird with a mass of 98 grams. (1 mark)

Solution
$s = 0.085(98) + 28.4 = 36.73$ cm
Specific behaviours
✓ calculates wing span

(d) Explain why it is difficult to comment on the reliability of the prediction in (c). (2 marks)

Solution
There is strong association between variables, which would indicate good reliability. However, as no original data is supplied, there is no way of telling if the prediction involves extrapolation, which is potentially unreliable. Hence difficult to comment.
Specific behaviours
✓ indicates no data to check whether extrapolation is involved ✓ mentions strength of association is good

(e) The mean mass of the birds in the sample was 84.8 grams. Determine the mean wing span of birds in the sample. (1 mark)

Solution
$s = 0.085(84.8) + 28.4 \approx 35.6$ cm
Specific behaviours
✓ calculates mean span

Question 19 **(8 marks)**

An art gallery records the value of all artworks at the start of each year for insurance purposes. The first valuation of a picture was \$4 800, and at the start of the next two years the picture was valued at \$5 040 and \$5 292 respectively.

(a) Show that the picture values form a geometric sequence. (2 marks)

Solution
$5040 \div 4800 = 1.05$ $5292 \div 5040 = 1.05$ Both ratios of terms same, which is consistent with geometric sequence
Specific behaviours
✓ calculates ratio using two pairs of values ✓ states ratios are the same and draws conclusion

(b) Assuming that the value of the picture continues to increase in this way,

(i) calculate the increase in value of the picture during the third year. (2 marks)

Solution
Increasing by 5% per year: $5292 \times 0.05 = \$264.60$
Specific behaviours
✓ states 5% increase per year OR calculates term 4 ✓ calculates increase

(ii) calculate the insurance premium for the picture in the tenth year, if the premium is 2.5% of the value of the picture. (2 marks)

Solution
$T_{10} = 4800(1.05)^{10-1} = 7446.38$ $7446.38 \times 0.025 = \$186.16$
Specific behaviours
✓ calculates value in tenth year ✓ calculates premium

(iii) determine the year in which the insurance premium, still 2.5% of the value of the picture, will first exceed \$300. (2 marks)

Solution
$T_n \times 0.025 = 300 \Rightarrow T_n = 12000$ $T_{19} \approx 11552,\ T_{20} \approx 12129$ The premium for the 20th year.
Specific behaviours
✓ calculates required value ✓ determines correct year